

$$\sqrt{x^2} = \sqrt{25}$$
$$x = \pm 5$$

$$x + 5 = 10$$
$$x = 5$$

$$\sqrt[x]{2^x} = 32$$

## Logarithmic Functions

$$y = 2^x$$

x	y
0	1
1	2
2	4
3	8
4	16

$$32 = 2^x$$

$$x = 5$$

x	y
5	32
6	64

$$50 = 2^x$$

$$\text{Log}_2 32 = x \leftrightarrow 2^x = 32$$

Definition

$$\text{Log}_a X = m \leftrightarrow a^m = X$$

$$\log_2 1 = x \leftrightarrow 2^x = 1$$

$$\log_5 \frac{1}{8} = x \leftrightarrow 5^x = \frac{1}{8}$$

$$\text{Log} \rightarrow \text{Log}_{10}$$

$$\text{Log } 100 = 2$$

$$\text{Log } .001 = -3$$

$$10^{-3} = \frac{1}{10^3} = \frac{1}{1000}$$

$$\text{Log } \frac{15}{8} = .273$$

$$\frac{\text{Log } 15}{\text{Log } 8} = 1.302$$

$$\text{Log}_{10} 10 = 1 \quad \leftrightarrow \quad 10^1 = 10$$

$$\text{Log}_3 3 = 1 \quad \leftrightarrow \quad 3^1 = 3$$

Rule  $\text{Log}_a a = 1$

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$$\text{Log} 1 = 0 \quad \leftrightarrow \quad 10^0 = 1$$

$$\text{Log}_3 1 = 0$$

Rule  $\text{Log}_a 1 = 0$

your turn  
solve

$$1) \log_3 81 = 4 \quad 3^x = 81 \quad 3^x = 3^4 \quad x = 4$$

$$2) \log_9 3 = \frac{1}{2} \quad 9^x = 3 \quad \sqrt{9} = 3 \quad 9^{\frac{1}{2}} = 3$$

$$3) \log 13400 = 4.127$$

$$4) \log_5 125 = \quad 5^x = 125 \rightarrow 5^x = 5^3 \\ x = 3$$

$$5) \log_2 \frac{1}{8} = \quad 2^x = \frac{1}{8} \rightarrow 2^x = \frac{1}{2^3} \rightarrow 2^x = 2^{-3} \\ x = -3$$

$$\text{Log}(20 \cdot 5) =$$

a)  $\text{Log}(20) \cdot \text{Log}(5)$

b)  $5 \cdot \text{Log}(20)$

c)  $\text{Log}(20) + \text{Log}(5)$

Rule

$$\text{Log}_a(MN) = \text{Log}_a M + \text{Log}_a N$$

$$\text{Log}_3(2x) = \text{Log}_3 2 + \text{Log}_3 x$$

$$\text{Log}_5(7 \cdot 7 \cdot 7) = \text{Log}_5 7 + \text{Log}_5 7 + \text{Log}_5 7$$

$$\text{Log}_5(7^3) \leftrightarrow 3 \text{Log}_5 7$$

Rule  $\text{Log}_a M^p = p \text{Log}_a M$

$$1) \text{Log}_a a^{-5} \rightarrow -5 \text{Log}_a a = -5$$

$$2) \text{Log}_7 \sqrt[3]{x} \rightarrow \text{Log}_7 (x)^{1/3} \rightarrow \frac{1}{3} \text{Log}_7 x$$

$$\text{Log } \frac{15}{8} = \text{Log } 15 - \text{Log } 8$$

Rule  $\text{Log}_a \frac{M}{N} = \text{Log}_a M - \text{Log}_a N$

$$\begin{aligned} 1) \text{ Log } \frac{5}{x^2} &= \text{Log } 5 - \text{Log } x^2 \\ &= \text{Log } 5 - 2\text{Log } x \end{aligned}$$

Change of Base formula

$$\text{Log}_b M = \frac{\text{Log}_a M}{\text{Log}_a b}$$

$$\text{Log}_2 32 = \frac{\text{Log } 32}{\text{Log } 2} = 5$$

$$\text{Log}_5 8 = \frac{\text{Log } 8}{\text{Log } 5} = 1.292$$

$$e = 2.71828 \dots$$

$$\text{Log}_e = \text{LN} \quad \leftarrow \begin{array}{l} \text{Natural} \\ \text{Log} \end{array}$$

$$\text{Log}_2 32 = \frac{\text{Log } 32}{\text{Log } 2} = \frac{\text{LN } 32}{\text{LN } 2}$$

$$\text{Log}_4 31 = \frac{\text{Ln } 31}{\text{Ln } 4} = 2.477$$

$$\begin{aligned}\text{Ln } 1900 &= 7.550 \\ &= \frac{\text{Log } 1900}{\text{Log } e}\end{aligned}$$

Solve  $7^{x-2} = 60$

$$\downarrow \text{Log}_7 (x-2) = \text{Log}_7 60$$

$$\frac{(x-2) \text{Log}_7}{\text{Log}_7} = \frac{\text{Log}_7 60}{\text{Log}_7}$$

$$x - 2 = \frac{\text{Log}_7 60}{\text{Log}_7}$$

$$x = \frac{\text{Log}_7 60}{\text{Log}_7} + 2$$

$$x = 4.104$$

$$\text{Log}_7 7^{x-2} = \text{Log}_7 60$$

$$(x-2) \text{Log}_7 7 = \text{Log}_7 60$$

$$x - 2 = \text{Log}_7 60$$

Solve  $e^{0.06x} = 1500$

$$\ln e^{0.06x} = \ln 1500$$

$$0.06x \ln e = \ln 1500$$

$$0.06x = \ln 1500$$

$$x = \frac{\ln 1500}{0.06}$$

$$\text{solve } 3^{x-1} = 7$$

$$\ln 3^{x-1} = \ln 7$$

$$(x-1) \ln 3 = \ln 7$$

$$x-1 = \frac{\ln 7}{\ln 3}$$

$$x = \frac{\ln 7}{\ln 3} + 1$$

$$\cancel{4} \log_4 (8x-6) = \cancel{4}^3$$

$$8x-6 = 4^3$$

$$8x-6 = 64$$

$$8x = 70$$

$$x = 8.75$$

$$\text{Log } (-2) = x$$

$$10^x = -2$$

$$\text{Log } x + \text{Log } (x-3) = 1$$

$${}_{10}\text{Log } x(x-3) = {}_{10}1$$

$$x^2 - 3x = 10$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$x = 5 \quad x = -2$$

$$\log x - \log(x+3) = 1$$

$$\log \frac{x}{x+3} = 1$$

$$\frac{x}{x+3} = \frac{10}{1}$$

$$x = 10(x+3)$$

$$x = 10x + 30$$

$$-9x = 30$$

$$x = \frac{-30}{9}$$

$$x = -3.\bar{3}$$